AKA:

6.2 Section Substitution (U Substitution)

Ex: $\int (3x+2)^2 \, dx$

Step 1 Set (u) = 3x+2 ; just chain rule piece
du = derivative of (u) = 3 dx

Step 2 now Δ every (x) to a (u) using the above values, so

$$= \int u^2 \frac{du}{3} = \frac{1}{3} \int u^2 du = \frac{1}{3} \cdot \frac{u^3}{3} + C$$

b/c du = 3dx
$\frac{du}{3} = dx$

Step 3 Have to return the (u) to an (x) in order to get final answer.

We know (u) is (3x+2), so

$$\boxed{= \frac{1}{9}(3x+2)^3 + C}$$

Ex: $\int \sqrt{5x+1}\, dx$

$\boxed{\text{Step 1}}$ Convert to a power

$= \int (5x+1)^{1/2}\, dx$

$\boxed{\text{Step 2}}$ Undo chain rule by setting chain rule piece $(5x+1)$ as any (u) value, so

$u = 5x+1$

du = derivative of (u) $= 5dx$

$\boxed{\text{Step 3}}$ = Connect pieces

$= \int u^{1/2} \cdot \frac{du}{5}$, so $\frac{1}{5}\int u^{1/2} du$

$\frac{du}{5} = dx$

$\boxed{\text{Step 4}}$ Integrate

$= \frac{1}{5} \cdot \frac{u^{1/2+1}}{3/2}$ or $\frac{1}{5} \cdot \frac{2}{3} u^{3/2} + C$

$\boxed{\text{Step 5}}$ Plug back in (x) value

$= \frac{2}{15}(5x+1)^{3/2} + C$

Using half-way mental substitution

Ex:

$$\int (3x+2)^2\,dx \qquad \begin{aligned} u &= 3x+2 \\ du &= 3\,dx \end{aligned} \quad \boxed{\text{Step 1}}$$

$\boxed{\text{Step 2}}$ $\boxed{\text{Step 4}}$

$\boxed{\text{Step 3}}$ $\rightarrow \dfrac{1}{3}\displaystyle\int 3(3x+2)^2\,dx = \dfrac{1}{3}\displaystyle\int (3x+2)^2\, 3\,dx$

my (3) that is missing

B/c we do not have a (3) how did it get cancelled out? by a $\frac{1}{3}$

$\boxed{\text{Step 5}}$ $= \dfrac{1}{3}\dfrac{(3x+2)^3}{3} + C$ now 3dx goes away

$\boxed{\text{Step 6}}$ Clean up!! $= \dfrac{1}{9}(3x+2)^3 + C$

Full mental look @ process

$$\int (3x+2)^2\,dx \qquad \begin{aligned} u &= 3x+2 \\ du &= \underline{3}dx \end{aligned}$$

$$= \frac{1}{3}\,\frac{(3x+2)^3}{3} + C = \boxed{\frac{1}{9}(3x+2)^3 + C}$$

from chain rule

Ex: $\int \sqrt{5x+1}\, dx$

Step 1 Convert to power

$\frac{1}{5}\int (5x+1)^{1/2}\, 5\,dx$

Step 3 add in my (5) and on outside, they cancel each other out.

Step 2 $u = 5x+1$

$du = 5\,dx$

Step 4 $= \frac{1}{5} \cdot \frac{2}{3}(5x+1)^{3/2} + C$

$$\boxed{= \frac{2}{15}(5x+1)^{3/2} + C}$$

#1 $\int 5(3x-1)^{2/3}\,dx = \frac{1}{3}\cdot 5\int (3x-1)^{2/3}\,\cancel{dx}\ 3dx$

$u=(3x-1)$; $du = 3dx$

$= \frac{5}{\cancel{3}}\cdot(3x-1)^{5/3}\cdot\frac{\cancel{3}}{\cancel{5}} + C = \boxed{(3x-1)^{5/3}+C}$

#2 $\int 6\sqrt{4x-3}\,dx = \int 6\cdot\frac{1}{4}(4x-3)^{1/2}\ \cancel{4dx}$

$u = 4x-3$; $du = 4dx$

$= \int \frac{6}{4}(4x-3)^{3/2}\cdot\frac{2}{3} + C = \boxed{(4x-3)^{3/2}+C}$

#3 $\int \frac{5}{2x-1}\,dx = 5(2x-1)^{-1}\,dx = 5\cdot\frac{1}{2}\int(2x-1)^{-1}\,\cancel{2dx}$

$u = 2x-1$

$du = 2dx$

$= \frac{5}{2}\,\frac{(2x-1)^{0}}{0} + C$ * Power Rule fails to work b/c dividing by zero is undefined, so

$u=2x-1$
$du=2dx$
$= 5\cdot\frac{1}{2}\int \frac{1}{2x-1}\,\cancel{2dx} = \boxed{\frac{5}{2}\cdot\ln|2x-1|+C}$

Ex: $\int e^{4x}\,dx = \frac{1}{4}\int e^{4x}\,\cancel{4dx} = \boxed{\frac{1}{4}e^{4x}+C}$

recheck answer by taking its derivative

$u = 4x$
$du = 4dx$

Ex: $\int xe^{x^2+3}\,dx = \frac{1}{2}\int e^{x^2+3}\,\cancel{2x\,dx}$

$u = x^2+3$
$du = 2\underbrace{x\,dx}$

If have this already - just add (2)
if no $x\,dx$, cannot

$$\boxed{= \frac{1}{2}e^{x^2+3}+C}$$

Ex: $\int (x^2+2x-3)^{15}\,(8(x+1))\,dx$

Step 1 What is main focus? Is there a power rule? If yes, start there first; let it be your (u).

$\frac{1}{2}\cdot 8\int (x^2+2x-3)^{15}\,\cancel{2(x+1)\,dx}$ Goes away

Part of derivative

$u = (x^2+2x-3)$ * what is being raised to power
$du = (2x+2)\,dx$
$= 2\underbrace{(x+1)\,dx}$

already above so, just add (2)

Final Step $\frac{4(x^2+2x-3)^{16}}{16}+C$

$$\boxed{= \frac{1}{4}(x^2+2x-3)^{16}+C}$$

Ex 8:

$$\int 12(8x-3)\sqrt{4x^2-3x+1}\,dx$$

Put on outside & leave alone ?

$$12\int (4x^2-3x+1)^{1/2} \quad \cancel{(8x-3)\,dx}$$

derivative for (u)

Part of chain rule

$u = 4x^2-3x+1$

$du = \underbrace{(8x-3)}_{\text{ready to go}}dx$

Focus on integrating $(4x^2-3x+1)^{1/2}$, so

$$= \overset{4}{\cancel{12}} \cdot (4x^2-3x+1)^{3/2} \cdot \frac{2}{\cancel{3}} + C$$

$$\boxed{= 8(4x^2-3x+1)^{3/2} + C}$$

Try on your own:

Example: ~~#46 Challenge~~ #47 $\int \left((2x-1)e^{2x^2} + xe^{x^2}\right)dx$

$\boxed{\text{Step 1}}$ Δ into 2 different problems, so

$$= \int (2x-1)e^{2x^2-2x}\,dx + \int xe^{x^2}\,dx$$

$u = 2x^2 - 2x$

$du = 4x - 2\,dx$

$= 2(2x-1)\,dx$

$u = x^2$

$du = 2x\,dx$

$$= \frac{1}{2}\int e^{2x^2-2x}\,\cancel{2(2x-1)dx} \qquad = \frac{1}{2}\int e^{x^2}\,\cancel{2x\,dx}$$

$$= \frac{1}{2}e^{2x^2-2x} \qquad = \frac{1}{2}e^{x^2}$$

* hold constant until the end

Final answer: $\boxed{\frac{1}{2}e^{2x^2-2x} + \frac{1}{2}e^{x^2} + C}$

#46 Challenge Question

$$\int \frac{e^{x/2}+e^{-x/2}}{e^{x/2}-e^{-x/2}}\,dx$$

Issue? Picking a (u). most likely would be denominator b/c is same as $\ln = \frac{1}{x}$

$$u = e^{x/2} - e^{-x/2}$$

$$du = \frac{1}{2}e^{x/2} + \frac{1}{2}e^{-x/2}\,dx$$

$$= \frac{1}{2}(e^{x/2}+e^{-x/2})\,dx$$

$$2\int \frac{1}{e^{x/2}-e^{-x/2}} \qquad \cancel{\frac{1}{2}(e^{x/2}+e^{-x/2})\,dx}$$

$$\boxed{= 2\ln|e^{x/2}-e^{-x/2}| + C}$$